

Equations and Relationships

MODULE



11



ESSENTIAL QUESTION

How can you use equations and relationships to solve real-world problems?



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Real-World Video

Businesses use equations to determine how many products can be made from the materials available. A guitar company uses an equation to determine how many guitars can be made with x amount of strings.

LESSON 11.1

Writing Equations to Represent Situations

 **CA CC** 6.EE.5, 6.EE.6, 6.EE.7

LESSON 11.2

Addition and Subtraction Equations

 **CA CC** 6.EE.5, 6.EE.6, 6.EE.7

LESSON 11.3

Multiplication and Division Equations

 **CA CC** 6.EE.5, 6.EE.6, 6.EE.7

LESSON 11.4

Writing Inequalities

 **CA CC** 6.EE.5, 6.EE.6, 6.EE.8

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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Evaluate Expressions

EXAMPLE Evaluate $8(3+2) - 5^2$

$$\begin{aligned} 8(3+2) - 5^2 &= 8(5) - 5^2 && \text{Perform operations inside parentheses first.} \\ &= 8(5) - 25 && \text{Evaluate exponents.} \\ &= 40 - 25 && \text{Multiply.} \\ &= 15 && \text{Subtract.} \end{aligned}$$

Evaluate the expression.

- $4(5 + 6) - 15$ _____
- $8(2 + 4) + 16$ _____
- $3(14 - 7) - 16$ _____
- $6(8 - 3) + 3(7 - 4)$ _____
- $10(6 - 5) - 3(9 - 6)$ _____
- $7(4 + 5 + 2) - 6(3 + 5)$ _____
- $2(8 + 3) + 4^2$ _____
- $7(14 - 8) - 6^2$ _____
- $8(2 + 1)^2 - 4^2$ _____

Connect Words and Equations

EXAMPLE The product of a number and 4 is 32.

The product of x and 4 is 32. *Represent the unknown with a variable.*

$4 \times x$ is 32.

Determine the operation.

$4 \times x = 32$.

Determine the placement of the equal sign.

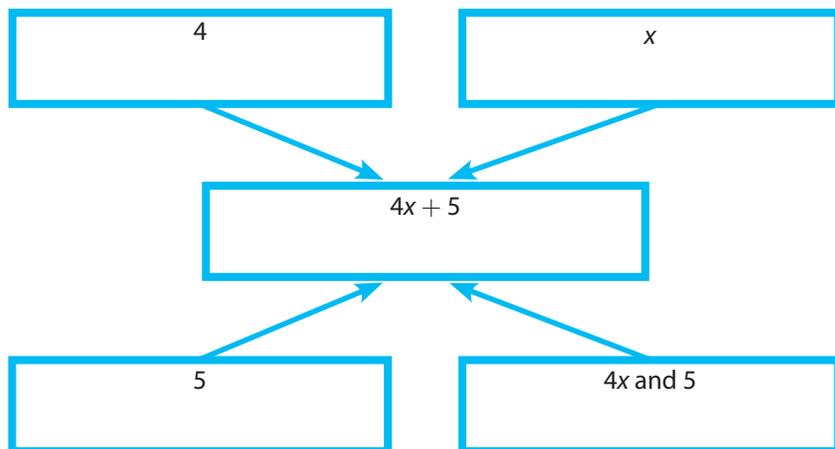
Write an algebraic equation for the word sentence.

- A number increased by 7.9 is 8.3. _____
- 17 is the sum of a number and 6. _____
- The quotient of a number and 8 is 4. _____
- 81 is three times a number. _____
- The difference between 31 and a number is 7. _____
- Eight less than a number is 19. _____

Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the graphic.



Understand Vocabulary

Match the term on the left to the correct expression on the right.

- | | |
|-------------------------|--|
| 1. algebraic expression | A. A mathematical statement that two expressions are equal. |
| 2. equation | B. A value of the variable that makes the statement true. |
| 3. solution | C. A mathematical statement that includes one or more variables. |

Vocabulary

Review Words

- ✓ algebraic expression (*expresión algebraica*)
- ✓ coefficient (*coeficiente*)
- ✓ constant (*constante*)
- evaluating (*evaluar*)
- like terms (*términos semejantes*)
- ✓ term (*término, en una expresión*)
- ✓ variable (*variable*)

Preview Words

- equation (*ecuación*)
- equivalent expression (*expresión equivalente*)
- properties of operations (*propiedades de las operaciones*)
- solution (*solución*)

Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.





GETTING READY FOR

Equations and Relationships

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

CA CC 6.EE.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Key Vocabulary

equation (*ecuación*)

A mathematical sentence that shows that two expressions are equivalent.

What It Means to You

You will learn to write an equation to represent a situation.

EXAMPLE 6.EE.7

The Falcons won their football game with a score of 30 to 19. Kevin scored 12 points for the Falcons. Write an equation to determine how many points Kevin's teammates scored.



Kevin's points	+	Teammates' points	=	Total points
12	+	t	=	30

CA CC 6.EE.5

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Key Vocabulary

inequality (*desigualdad*)

A mathematical sentence that shows the relationship between quantities that are not equal.

What It Means to You

You can substitute a given value for the variable in an equation or inequality to check if that value makes the equation or inequality true.

EXAMPLE 6.EE.5

Melanie has to buy 6 tickets to a play. She will pay at least \$156 depending on the price of the seats. Write an inequality for this situation. Use it to decide if \$20 and \$30 are possible ticket prices.

Number of tickets bought	.	Price per ticket	\geq	Total cost
6	.	p	\geq	156

Substitute 20 and 30 for p to see if the inequality is true.

$6p \geq 156$	$6p \geq 156$
$6 \cdot 20 \stackrel{?}{\geq} 156$	$6 \cdot 30 \stackrel{?}{\geq} 156$
$120 \stackrel{?}{\geq} 156 \times$	$180 \stackrel{?}{\geq} 156 \checkmark$

The price per ticket could be \$30 but not \$20.



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LESSON 11.1 Writing Equations to Represent Situations

 CA CC 6.EE.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q , and x are all nonnegative rational numbers. Also 6.EE.5, 6.EE.6



ESSENTIAL QUESTION

How do you write equations and determine whether a number is a solution of an equation?

Determining Whether Values Are Solutions

An **equation** is a mathematical statement that two *expressions* are equal. An equation may or may not contain variables. For an equation that contains a variable, a **solution** of the equation is a value of the variable that makes the equation true.

An expression represents a single value.

An equation represents a relationship between two values.

	Expression	Equation
Numerical	$5 + 4$	$5 + 4 = 9$
Words	a number <i>plus</i> 4	A number <i>plus</i> 4 <i>is</i> 9.
Algebraic	$n + 4$	$n + 4 = 9$

An equation relates two expressions using a symbol for "is" or "equals."



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EXAMPLE 1

 CA CC 6.EE.5

Determine whether the given value is a solution of the equation.

A $x + 9 = 15$; $x = 6$

$6 + 9 \stackrel{?}{=} 15$ Substitute 6 for x .

$15 \stackrel{?}{=} 15$ Add.

6 is a solution of $x + 9 = 15$.

B $\frac{y}{4} = 32$; $y = 8$

$\frac{8}{4} \stackrel{?}{=} 32$ Substitute 8 for y .

$2 \stackrel{?}{=} 32$ Divide.

8 is not a solution of the equation $\frac{y}{4} = 32$.

C $8x = 72$; $x = 9$

$8(9) \stackrel{?}{=} 72$ Substitute 9 for x .

$72 \stackrel{?}{=} 72$ Multiply.

9 is a solution of $8x = 72$.

Math Talk



Mathematical Practices

Does the variable in each equation in parts **A–D** refer to a single number? Explain.



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YOUR TURN

Determine whether the given value is a solution of the equation.

1. $11 = n + 6; n = 5$

2. $y - 6 = 24; y = 18$

3. $\frac{x}{9} = 4; x = 36$

4. $15t = 100; t = 6$



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Writing Equations to Represent Situations

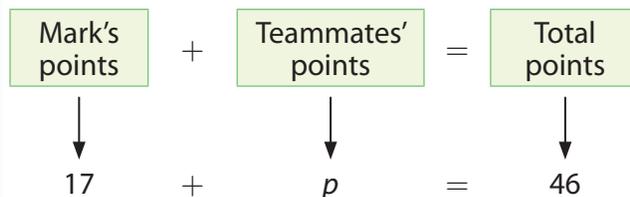
You can represent some real-world situations with an equation. Making a model first can help you organize the information and identify what the variable represents.

EXAMPLE 2



CA CC 6.EE.6

Mark scored 17 points for the home team in a basketball game. His teammates as a group scored p points. Write an equation to represent this situation.



p represents the number of points Mark's teammates scored.

YOUR TURN

Write an equation to represent each situation.

5. The sum of a number f and 9 is equal to 38.

6. The product of 17 and a number n is equal to 102.

7. Craig is c years old. His 12-year-old friend Kaitlin is $\frac{1}{2}$ year younger than Craig.

8. Kim rented skates for h hours. The rental fee was \$2.50 per hour. She paid a total of \$20.



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Writing an Equation and Checking Solutions

You can substitute a given value for the variable in a real-world equation to check if that value makes sense for the situation.



EXAMPLE 3



CA CC 6.EE.7

Sarah used a gift card to buy \$47 worth of food. She has \$18 left on her gift card. Write an equation to represent this situation, and use it to determine whether Sarah had \$65 or \$59 on the gift card before buying food.

STEP 1 Write a word equation based on the situation.

Amount on card	−	Amount spent	=	Amount left on card
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STEP 2 Rewrite the equation using a variable for the unknown quantity and the given values for the known quantities.

Let x be the amount on the card.

Amount on card	−	Amount spent	=	Amount left on card
x	−	47	=	18

The amount spent and the amount left on the card are the known quantities. Substitute those values in the equation.

STEP 3 Substitute 65 and 59 for x to see which equation is true.

$x - 47 = 18$	$x - 47 = 18$
$65 - 47 \stackrel{?}{=} 18$	$59 - 47 \stackrel{?}{=} 18$
$18 \stackrel{?}{=} 18$	$12 \stackrel{?}{=} 18$

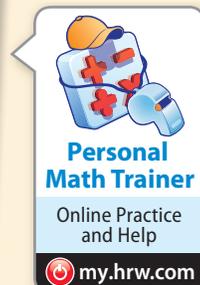
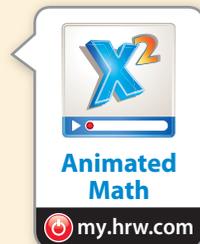
○ The amount on Sarah's gift card before she bought food was \$65.

Reflect

9. **What If?** Suppose Sarah had \$12 left on her gift card. How would this change the equation and the final answer?

YOUR TURN

10. On Saturday morning, Owen earned \$24. By the end of the afternoon he had earned a total of \$62. Write an equation to represent the situation. Tell what the variable represents. Determine whether Owen earned \$38 or \$31 on Saturday afternoon.



Guided Practice

Determine whether the given value is a solution of the equation. (Example 1)

- $23 = x - 9; x = 14$ _____
- $\frac{n}{13} = 4; n = 52$ _____
- $25 = \frac{k}{5}; k = 5$ _____
- $2.5n = 45; n = 18$ _____
- $21 - h = 15; h = 6$ _____
- $d - 4 = 19; d = 15$ _____
- $w - 9 = 0; w = 9$ _____
- $5q = 31; q = 13$ _____
- Each floor of a hotel has r rooms. On 8 floors, there are a total of 256 rooms. Write an equation to represent this situation. (Example 2)

Number	×	Number of rooms	=		_____
_____		_____		_____	

In Exercises 10–12, write an equation to represent the situation. (Example 2)

- When a number n is divided by 16, the result is 3. _____
- The sum of 4.5 and a number k is 26.2. _____
- In the school band, there are 5 trumpet players and f flute players. There are twice as many flute players as there are trumpet players. _____

In Exercises 13–14, write an equation to solve the problem. Tell what your variable represents. (Example 3)

- Pedro bought 8 tickets to a basketball game. He paid a total of \$208. Determine whether each ticket cost \$26 or \$28.

- The high temperature was 92 °F. This was 24 °F higher than the overnight low temperature. Determine whether the low temperature was 62 °F or 68 °F.



ESSENTIAL QUESTION CHECK-IN

- Tell how you can determine whether a number is a solution of an equation.

11.1 Independent Practice



CA CC 6.EE.5, 6.EE.6, 6.EE.7



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In Exercises 16–18, write an equation to solve the problem. Tell what your variable represents.

16. Andy's grandfather, who is 76 years old, is 4 times as old as Andy. Determine whether Andy is 19 or 22 years old.

17. Mi's sleeping bag weighs 8 pounds. Her backpack and sleeping bag together weigh 31 pounds. Determine whether her backpack alone weighs 25 or 23 pounds.

18. Halfway through a bus route, 23 students have been dropped off and 48 students remain on the bus. Determine whether there were 61 or 71 students on the bus to begin with.

19. Write an equation that involves multiplication, contains a variable, and has a solution of 5. Then write another equation that has the same solution and includes the same variable and numbers but uses division.

20. **Vocabulary** How are expressions and equations different?

21. **Multistep** Alan has partially completed a table showing the distances between his town, Greenville, and two other towns.

Distance between Greenville and Nearby Towns (miles)	
Parker	29
Hadley	?

- a. The distance between Hadley and Greenville is $12\frac{1}{2}$ miles less than the distance between Parker and Greenville. Write two equations that compare the distance between Hadley and Greenville and the distance between Parker and Greenville. Tell what your variable represents.

- b. Alan says the distance from Hadley to Greenville is $16\frac{1}{2}$ miles. Is he correct? Explain.

22. **Explain the Error?** A problem states that Ursula earns \$9 per hour. To write an expression that tells how much money Ursula earns for h hours, Joshua wrote $\frac{9}{h}$. Sarah wrote $9h$. Whose expression is correct and why?

- 23. Communicate Mathematical Ideas** A dog weighs 44 pounds. The dog needs to lose 7 pounds to reach its ideal weight. Mikala wrote the equation $x + 7 = 44$ to represent the situation. Kirk wrote the equation $44 - x = 7$. What does the variable represent? Which equation is correct? Can you write another equation that represents the situation?
-
-

- 24. Multiple Representations** The table shows the ages of Cindy and her dad.

Dad's Age	Cindy's Age
28 years old	2 years old
36 years old	10 years old
?	18 years old

- a.** Write an equation that relates Cindy's age to her dad's age when Cindy is 18. Tell what the variable represents.
-

- b.** Determine if 42 is a solution to the equation. Show your work.
-

- c.** Explain the meaning of your answer in part **b**.
-



FOCUS ON HIGHER ORDER THINKING

- 25. Critical Thinking** In the school band, there are 4 trumpet players and f flute players. The total number of trumpet and flute players is 12. Are there twice as many flute players as trumpet players? Explain.
-
-

- 26. Problem Solving** Ronald paid \$162 for 6 tickets to a basketball game. During the game he noticed that his friend paid \$130 for 5 tickets. The price of each ticket was \$26. Was Ronald overcharged? Explain.
-
-

- 27. Communicate Mathematical Ideas** Tariq said you can write an equation by setting an expression equal to itself. Would an equation like this be true? Explain.
-
-
-

Work Area

LESSON 11.2 Addition and Subtraction Equations

 **CA CC** 6.EE.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers. Also 6.EE.5, 6.EE.6



ESSENTIAL QUESTION

How do you solve equations that contain addition or subtraction?

EXPLORE ACTIVITY



 **CA CC** 6.EE.6, 6.EE.7

Modeling Equations

A puppy weighed 6 ounces at birth. After two weeks, the puppy weighed 14 ounces. How much weight did the puppy gain?

Let x represent the number of ounces gained.

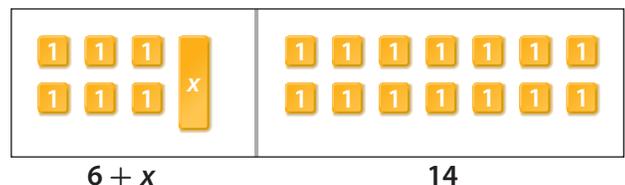
Weight at birth	+	Weight gained	=	Weight after 2 weeks
↓		↓		↓
6	+	x	=	14



To answer this question, you can solve the equation $6 + x = 14$.

Algebra tiles can model some equations. An equation mat represents the two sides of an equation. To solve the equation, remove the same number of tiles from both sides of the mat until the x tile is by itself on one side.

- A** Model $6 + x = 14$.
- B** How many 1 tiles must you remove on the left side so that the x tile is by itself? _____
Cross out these tiles on the equation mat.
- C** Whenever you remove tiles from one side of the mat, you must remove the same number of tiles from the other side of the mat. Cross out the tiles that should be removed on the right side of the mat.
- D** How many tiles remain on the right side of the mat? _____
This is the solution of the equation.
The puppy gained _____ ounces.



Math Talk

 **Mathematical Practices**

Why did you remove tiles from each side of your model?

Reflect

- 1. Communicate Mathematical Ideas** How do you know when the model shows the final solution? How do you read the solution?



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Using Subtraction to Solve Equations

Removing the same number of tiles from each side of an equation mat models subtracting the same number from both sides of an equation.

Subtraction Property of Equality

You can subtract the same number from both sides of an equation, and the two sides will remain equal.

When an equation contains addition, solve by subtracting the same number from both sides.

EXAMPLE 1



Solve the equation $a + 15 = 26$. Graph the solution on a number line.

$a + 15 = 26$ Notice that the number 15 is added to a .

$$\begin{array}{r} a + 15 = 26 \\ -15 \quad -15 \\ \hline a \quad = 11 \end{array}$$

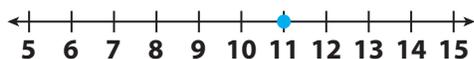
Subtract 15 from both sides of the equation.

Check: $a + 15 = 26$

$11 + 15 \stackrel{?}{=} 26$ Substitute 11 for a .

$26 \stackrel{?}{=} 26$ Add on the left side.

Graph the solution on a number line.

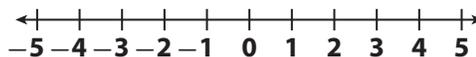


Reflect

2. **Communicate Mathematical Ideas** How do you decide which number to subtract from both sides?

YOUR TURN

3. Solve the equation $5 = w + 1\frac{1}{2}$.



Graph the solution on a number line.

$w =$ _____



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Solving Equations that Represent Geometric Concepts

You can write equations to represent geometric relationships.

Recall that a straight line has an angle measure of 180° . Two angles whose measures have a sum of 180° are called supplementary angles. Two angles whose measures have a sum of 90° are called complementary angles.

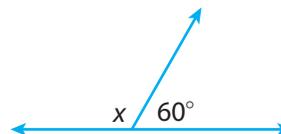
EXAMPLE 3



CA CC 6.EE.7, 6.EE.6

Find the measure of the unknown angle.

STEP 1 Write the information in the boxes.



Unknown angle measure	+	60°	=	180°
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STEP 2 Write a description to represent the model. Include a question for the unknown angle measure.

The sum of the measure of an unknown angle and 60° is 180° . What is the measure of the unknown angle?

STEP 3 Write an equation.

$$x + 60 = 180$$

x represents the measure of the unknown angle in degrees.

STEP 4 Solve the equation.

$$x + 60 = 180$$

$$\begin{array}{r} -60 \\ \hline \end{array}$$

$$x = 120$$

Subtract 60 from each side.

The final answer includes units of degrees.

The unknown angle measures 120° .

YOUR TURN

6. Write and solve an equation to find the measure of the unknown angle. Tell what the variable represents.

7. Write and solve an equation to find the complement of a 42° angle. Tell what the variable represents.



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Writing Real-World Problems for a Given Equation

You can write a real-world problem for a given equation. Examine each number and mathematical operation in the equation.



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EXAMPLE 4



6.EE.7

Write a real-world problem for the equation $21.79 + x = 25$. Then solve the equation.

STEP 1 Examine each part of the equation.

x is the unknown or quantity we are looking for.

21.79 is added to x .

$= 25$ means that after adding 21.79 and x , the result is 25 .



STEP 2 Write a real-world situation that involves *adding* two quantities.

Joshua wants to buy his mother flowers and a card for Mother's Day. Joshua has \$25 to spend and selects roses for \$21.79. How much can he spend on a card? Let x represent that amount.

STEP 3 Solve the equation.

$$\begin{array}{r} 21.79 + x = 25 \\ \underline{-21.79} \quad \underline{-21.79} \\ x = 3.21 \end{array}$$

The final answer includes units of money in dollars.

Joshua can spend \$3.21 on a Mother's Day card.

Math Talk



Mathematical Practices

How is the question in a real-world problem related to its equation?

Reflect

8. **What If?** How might the real-world problem change if the equation were $x - 21.79 = 25$ and you still used roses for \$21.79?

YOUR TURN

9. Write a real-world problem for the equation $x - 100 = 40$. Tell what the variable represents. Then solve the equation.



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Guided Practice

1. A total of 14 guests attended a birthday party. Three friends stayed after the party to help clean up. How many left when the party ended?
(Explore Activity)

a. Let x represent the _____

b.

 +

 =

_____ + _____ = _____

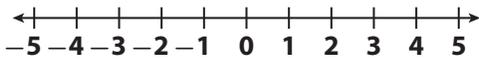
- c. Draw algebra tiles to model the equation.
_____ friends left when the party ended.



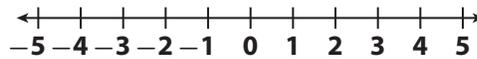
Solve each equation. Graph the solution on a number line.

(Examples 1 and 2)

2. $2 = x - 3$ $x =$ _____



3. $s + 12.5 = 14$ $s =$ _____



Solve each equation. (Examples 1 and 2)

4. $h + 6.9 = 11.4$

$h =$ _____

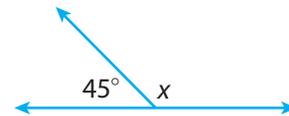
5. $82 + p = 122$

$p =$ _____

6. $n + \frac{1}{2} = \frac{7}{4}$

$n =$ _____

7. Write and solve an equation to find the measure of the unknown angle. Tell what the variable represents. (Example 3)



8. Write a real-world problem for the equation $x - 75 = 200$. Tell what the variable represents. Then solve the equation. (Example 4)



ESSENTIAL QUESTION CHECK-IN

9. How do you solve equations that contain addition or subtraction?

11.2 Independent Practice



CA CC 6.EE.5, 6.EE.6, 6.EE.7



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In 10–16, write and solve an equation to answer each question. Tell what the variable represents.

10. A wildlife reserve had 8 elephant calves born during the summer and now has 31 total elephants. How many elephants were in the reserve before summer began?

11. My sister is 14 years old. My brother says that his age minus 12 is equal to my sister's age. How old is my brother?

12. Kim bought a poster that cost \$8.95 and some colored pencils. The total cost was \$21.35. How much did the colored pencils cost?

13. Acme Cars sold 37 vehicles in June. How many compact cars were sold in June?

Acme Cars – June Sales	
Type of car	Number sold
SUV	8
Compact	?

14. Sandra wants to buy a new MP3 player that is on sale for \$95. She has saved \$73. How much more money does she need?

15. Ronald spent \$123.45 on school clothes. He counted his money and discovered that he had \$36.55 left. How much money did he originally have?

16. Brita withdrew \$225 from her bank account. After her withdrawal, there was \$548 left in her account. How much money did Brita have in her account before the withdrawal?

17. Represent Real-World Problems Write a real-world situation that can be represented by $15 + c = 17.50$. Tell what the variable represents. Then solve the equation and describe what your answer represents for the problem situation.

18. Critique Reasoning Paula solved the equation $7 + x = 10$ and got 17. She is not certain that her answer is correct. How could you explain her mistake to her?



19. Multistep A grocery store is having a sale this week. If you buy a 5-pound bag of apples for the regular price, you can get another bag for \$1.49. If you buy a 5-pound bag of oranges at the regular price, you can get another bag for \$2.49.

Product	Regular price
5-pound bag of apples	\$2.99
5-pound bag of oranges	\$3.99

a. Write an equation to find the discount for each situation using a for the amount of the discount for apples and r for the amount of the discount for oranges.

b. Which fruit has a greater discount? Explain.

20. Critical Thinking An orchestra has twice as many woodwind instruments as brass instruments. There are a total of 150 brass and woodwind instruments.

a. Write two different addition equations that describe this situation. Use w for the number of woodwinds and b for the number of brass instruments.

b. How many woodwinds and how many brass instruments must there be to satisfy the given information?

21. Look for a Pattern Assume the following: $a + 1 = 2$, $b + 10 = 20$, $c + 100 = 200$, $d + 1,000 = 2,000$, ...

a. Solve each equation.

b. What pattern do you notice in the values of the variables?

c. What would be the value of g if the pattern continued?
